Precalc Honor 11-4 Roots of Complex Numbers Name: _____________

1. Find the three cube roots of -1.

We know the real cube root of -1. It's -1. But there are two more complex cube roots of -1.

There is some complex number, z, that when cubed equals -1:

$$z^3 = -1$$
$$(r cis\theta)^3 = 1cis180^{\circ}$$
$$r^3 cis 3\theta = 1cis180^{\circ}$$

Separate the r's and θ's:

$r^3 = 1$ and $3\theta = 180^{\circ} + 360^{\circ}n$

so $r = 1$ and $\theta = 60^{\circ} + 120^{\circ}n$

cube root #1: $1cis60^{\circ}$

cube root #2: $1cis180^{\circ}$

cube root #3: $1cis300^{\circ}$

Confirm that when these roots are cubed the answer is -1.

"Roots of a Complex Number" Theorem: There are n distinct nth roots of z=rcis θ. They are

$$z^{\frac{1}{n}} = r^{\frac{1}{n}} cis\left(\frac{\theta}{n} + \frac{360^{\circ}}{n}k\right)$$ for k = 0, 1, …, n-1

In the example above: $(1cis180^{\circ})^{\frac{1}{3}} = 1^{\frac{1}{3}}cis(\frac{180^{\circ}}{3} + \frac{360^{\circ}}{3})$

Graphing Observation: For n>2, the n^{th} roots of a complex number form:

2. Find the cube roots of 8i and graph them on the Argand plane (the complex plane). What shape do the roots seem to form in the Argand plane?

3. Find the 4th roots of $\left(8\sqrt{3} + 8i\right)$ and graph them on the Argand plane. What shape do the roots seem to form in the polar coordinate plane?

4. Find the square roots of -2+2i $\sqrt{3}$. Plot both -2+2i $\sqrt{3}$ and its square roots on the Argand plane.