Precalc Honor | 11-3 DeMoivre's Theorem | Name: _____________

Raising a Complex Number to a Power

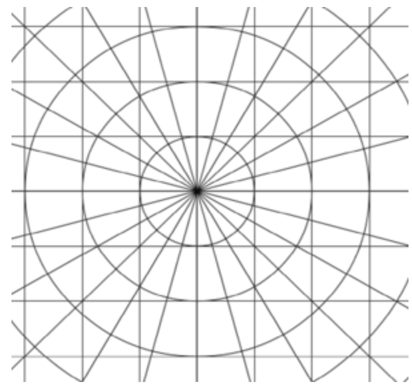
DeMoivre's Theorem:

If a complex number, $z \;=\; r(cos\theta + isin\theta) \;=\; rcis\theta$ is raised to an integer power, n,

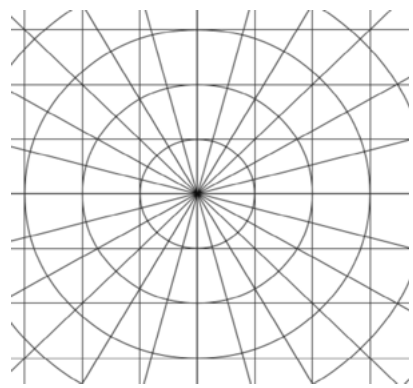
then $z^n = r^n\big(\cos(n\theta) + isin(n\theta)\big) = r^n cis(n\theta)$

1. Change each complex number into polar form, then find each of the following.

a. $(1+\sqrt{3}i)^2$



b. $(2+2i)^4(1+\sqrt{3}i)^2$



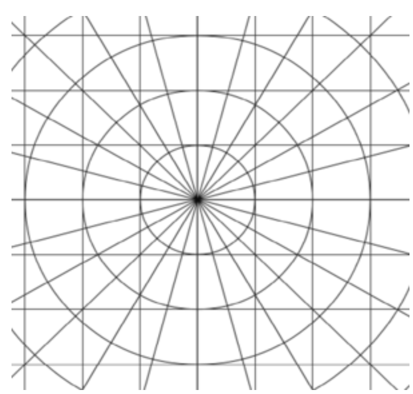
2. Let z=-1. Use DeMoivre's theorem to show that the positive powers of z are alternately ± 1. Remember that $z = -1 = -1 + 0i$. Change this to polar form and find the following powers of z:

$z^1 =$

$z^2 =$

$z^3 =$

$z^4 =$



3. Let $z = 2\text{cis}\frac{\pi}{4}$. Calculate z^n for n= 1, 2, 3, 4, 5.

$z^1 =$

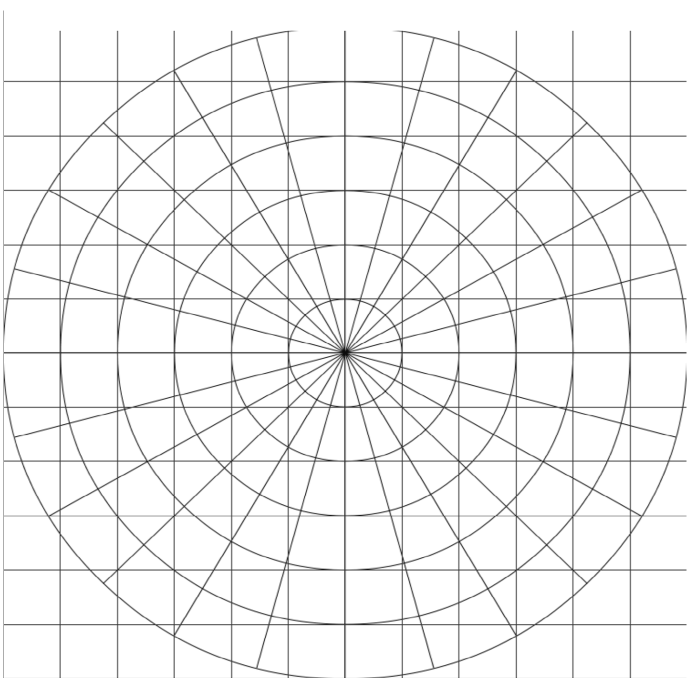
$z^2 =$

$z^3 =$

$z^4 =$

$z^5 =$

b. Plot these powers of z. What do you notice?



4. Dividing Complex Numbers

a. $\frac{6cos60^{\circ}+isin60^{\circ}}{3cos90^{\circ}+3sin90^{\circ}}$

b. $\frac{3+3i}{cos90^{\circ}+isin90^{\circ}}$

To divide complex numbers:

$$\frac{z_1}{z_2} = \frac{r_1(\cos(\theta_1) + isin\,(\theta_1))}{r_2(\cos(\theta_2) + isin\,(\theta_2))} = \frac{r_1}{r_2}(\cos(\theta_1 - \theta_2) + isin\,(\theta_1 - \theta_2))$$