Precalc Honor 11-2 Graphing, Multiplying Complex Numbers Name: _____

I. Multiply Complex Numbers in Rectangular Form:

a. $(2 + i\sqrt{3})(2 + i\sqrt{3})$

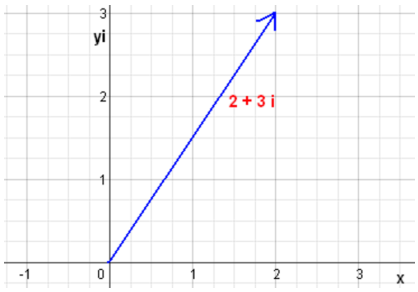
b. $(2 + i\sqrt{3})(2 - i\sqrt{3})$

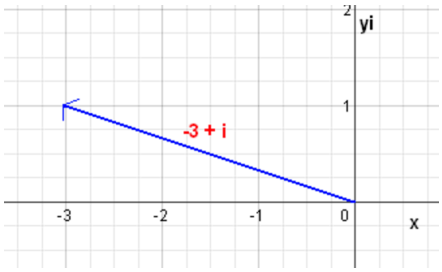
II. Divide Complex Numbers:

$\frac{2+3i}{1-4i}$ Use complex conjugate to write the quotient in $a + bi$ form.

III. Graphing Complex Numbers

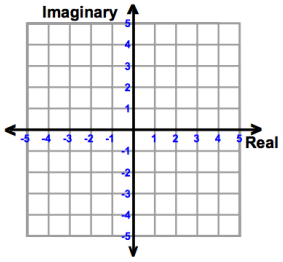
Complex numbers can be represented by vectors in a Cartesian plane.

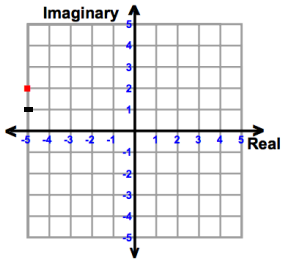




The real part of the complex number, a, is represented by the x coordinate, and the imaginary part of the complex number, b, is represented by the y coordinate.

-2 + 5i





IV. Absolute Value of a Complex Number, "Modulus"

The absolute value of a complex number length of its vector:

Example:

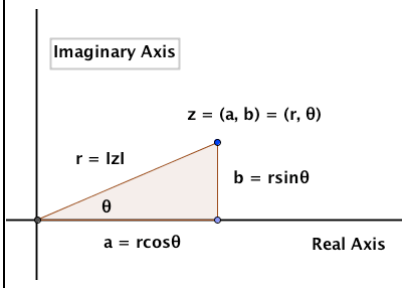
$i = \sqrt{-1}$

Complex Number:
$Z = a + bi$

a = real part
b = imaginary part

Complex Conjugates:
$a \pm bi$

$(a + bi)(a - bi) =$
$real\ number$



Rectangular Coordinates:
(a, b)
Rectangular Form:
$a + b$

Polar Coordinates:
$(rcos\theta, rsin\theta)$
Polar Form:
$rcos\theta + rsin\theta \cdot i$

$|z| = \sqrt{a^2 + b^2}$

V. Converting Between Polar and Rectangular Form

Convert to polar form:

$1 - i\sqrt{3}$

Convert to rectangular form:

$\sqrt{2}cos45^\circ + \sqrt{2}sin45^\circ$

$\sqrt{2}\left(\cos 45^\circ + i\sin 45^\circ\right)$

$\sqrt{2}cis45^\circ$ abbreviation

Rectangular Form:

$a + b$

Polar Form:

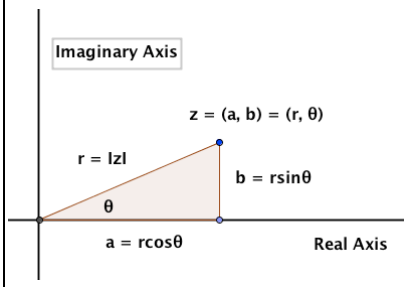
$rcos\theta + rsin\theta \cdot i$

$r\cos\theta + (r\sin\theta)i$

$r(\cos\theta + i\sin\theta)$

$rcis\theta$

VI. Multiplication in Rectangular Form vs. Polar Form



Multiply the following complex numbers using the FOIL method:

$(\sqrt{2} + i\sqrt{2})(-3\sqrt{2} + 3i\sqrt{2}) =$

Convert your product above to polar form.

Convert each of the original complex numbers to polar form.

Compare the complex numbers to their product in polar form. Do you notice anything?

Multiplying Two Complex Numbers in Polar Form

1. Multiply their r values
2. Add their angles

$$z_1 = r_1 cis\theta_1$$

$$z_2 = r_2 cis\theta_2$$

$$z_1 \cdot z_2 = r_1 cis\theta_1 \cdot r_2 cis\theta_2 = r_1 r_2 cis(\theta_1 + \theta_2)$$

$$\boldsymbol{z_1 z_2 = r_1 r_2 cis(\theta_1 + \theta_2)}$$

Multiply and then convert your answer to rectangular form.

(3 cis 165°)(4 cis 45°)

$$z_1 z_2 = r_1 r_2 cis(\theta_1 + \theta_2)$$

Which operation would you rather perform?

$(2 + 2i)^6$ or $(2\sqrt{2}cis45^\circ)^6$