Precalc Honor | 11-4 Roots of Complex Numbers | Name: ______________

1. Find the three cube roots of -1.

$(-1)^3 = -1, \quad \sqrt[3]{-1} = -1$

We know the real cube root of -1. It's -1. But there are two more complex cube roots of -1.

There is some complex number, z, that when cubed equals -1:

$-1 + 0i$

$$z^3 = -1$$

$$(r cis\theta)^3 = 1 cis 180^\circ$$

$$r^3 cis 3\theta = 1 cis 180^\circ$$

Separate the r's and θ's:

$r^3 = 1$ and $3\theta = 180^\circ + 360^\circ n$

so $r = 1$ and $\theta = 60^\circ + 120^\circ n$

cube root #1: $1 cis 60^\circ$

cube root #2: $1 cis 180^\circ$

cube root #3: $1 cis 300^\circ$

Confirm that when these roots are cubed the answer is -1.

$(1 cis 60^\circ)^3 = 1^3 cis(3 \cdot 60^\circ) = 1 cis 180^\circ$

$(1 cis 180^\circ)^3 =$

$(1 cis 300^\circ)^3 = 1^3 cis(3 \cdot 300^\circ) = 1 cis 900^\circ$

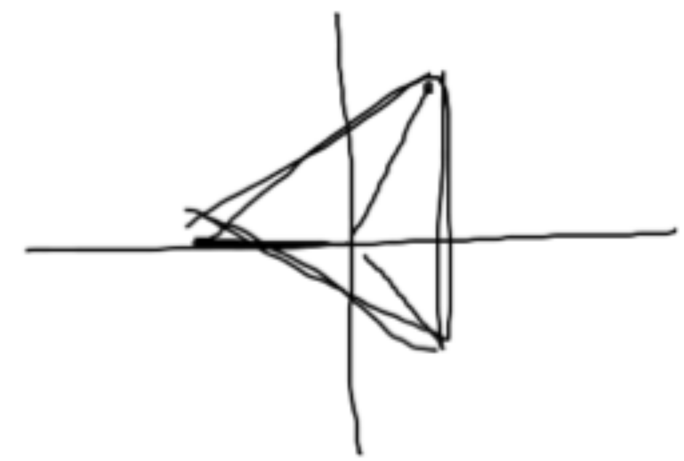
$1 cis 180^\circ$

"Roots of a Complex Number" Theorem: There are n distinct nth roots of z=rcis θ. They are

$$z^{\frac{1}{n}} = r^{\frac{1}{n}} cis\left(\frac{\theta}{n} + \frac{360^\circ}{n}k\right) \qquad \text{for } k = 0, 1, \ldots, n-1$$

In the example above: $(1 cis 180^\circ)^{\frac{1}{3}} = 1^{\frac{1}{3}} cis(\frac{180^\circ}{3} + \frac{360^\circ}{3})$

Graphing Observation: For n>2, the n^{th} roots of a complex number form:



$1 \text{ cis } 60°$

$1 \text{ cis } 180°$

$1 \text{ cis } 300°$

2. Find the cube roots of 8i and graph them on the Argand plane (the complex plane). What shape do the roots seem to form in the Argand plane?

$0 + 8i \longrightarrow r \text{ cis } \theta$

$z^3 = 8 \text{ cis } 90°$

$8^{1/3} \text{ cis}\left(\frac{90°}{3} + \frac{360°}{3}\right)$ add 120°

$2 \text{ cis } 30°$

$2 \text{ cis } 150°$

$2 \text{ cis } 270°$

check:

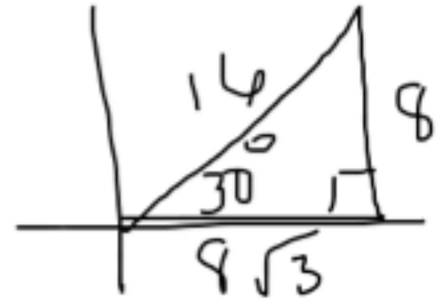
$(2 \text{ cis } 30)^3 = 8 \text{ cis } 90°$

$(2 \text{ cis } 150)^3 = 8 \text{ cis } 450° = 8 \text{ cis } 90°$

$(2 \text{ cis } 270°)^3 = 8 \text{ cis } 810° = 8 \text{ cis } 90°$

3. Find the 4[th] roots of $(8\sqrt{3}+8i)$ and graph them on the Argand plane. What shape do the roots seem to form in the polar coordinate plane?

$(16 \operatorname{cis} 30^\circ)^{1/4}$

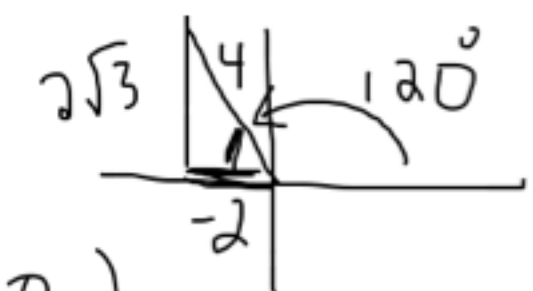


$16^{1/4} \operatorname{cis}\left(\frac{30}{4} + \frac{360}{4}\right)$ add 90°

$2 \operatorname{cis} 7.5^\circ$
$2 \operatorname{cis} 97.5^\circ$
$2 \operatorname{cis} 187.5^\circ$
$2 \operatorname{cis} 277.5^\circ$

4. Find the square roots of -2+2i $\sqrt{3}$. Plot both -2+2i $\sqrt{3}$ and its square roots on the Argand plane

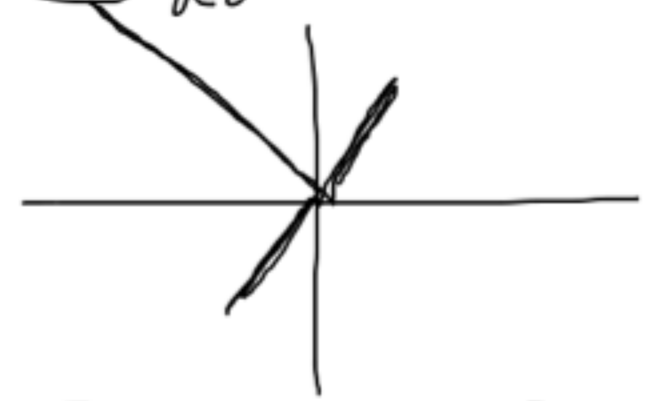
$(4 \operatorname{cis} 120^\circ)^{1/2}$



$4^{1/2} \operatorname{cis}\left(\frac{120}{2} + \frac{360}{2}\right)$ add 180°

$2 \operatorname{cis} 60^\circ$

$2 \operatorname{cis} 240^\circ$



check

$(2 \operatorname{cis} 240)^2 = 4 \operatorname{cis} 480^\circ = 4 \operatorname{cis} 120^\circ$