Precalc Honor    11-3 DeMoivre's Theorem    Name: ______________

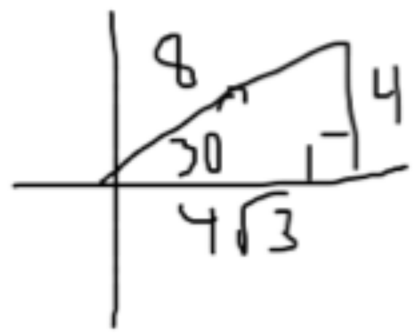
1. Change each complex number into polar form, then find each of the following. Look for patterns!

a. $(4\sqrt{3}+4i)^2$ $= (8 \text{ cis } 30^\circ)^2 = (8 \text{ cis } 30^\circ)(8 \text{ cis } 30^\circ)$

$= r \text{ cis } \theta$

$= 64 \text{ cis } 60^\circ$

$= r^2 \text{ cis}(2\theta)$




b. $(4\sqrt{3}+4i)^3$ $= (8 \text{ cis } 30^\circ)^3 = \underbrace{(8 \text{ cis } 30^\circ)(8 \text{ cis } 30^\circ)}(8 \text{ cis } 30^\circ)$

$= (64 \text{ cis } 60^\circ)(8 \text{ cis } 30^\circ)$

$= 512 \text{ cis } 90^\circ$

$= 8^3 \text{ cis}(3 \cdot 30^\circ)$

$= r^3 \text{ cis}(3\theta)$

c. $(4\sqrt{3}+4i)^n$

$(8 \text{ cis } 30^\circ)^n = 8^n \text{ cis}(n \cdot 30^\circ)$

DeMoivre's Theorem: If $\underbrace{z=r(\cos\theta + i\sin\theta)}_{r \text{ cis } \theta}$ and n is an integer, then

$z^n = r^n \text{ cis}(n \cdot \theta)$

2. Let z= $1.5\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ . Calculate $z^n$ for n= 1, 2, 3, 4, 5.

$z = 1.5 \operatorname{cis} \pi/4$

$z^1 = 1.5 \operatorname{cis} \pi/4$

$z^2 = (1.5)^2 \operatorname{cis}(2 \cdot \pi/4) = 2.25 \operatorname{cis} \frac{\pi}{2}$

$z^3 = (1.5)^3 \operatorname{cis}(3 \cdot \pi/4) = 3.375 \operatorname{cis} \frac{3\pi}{4}$

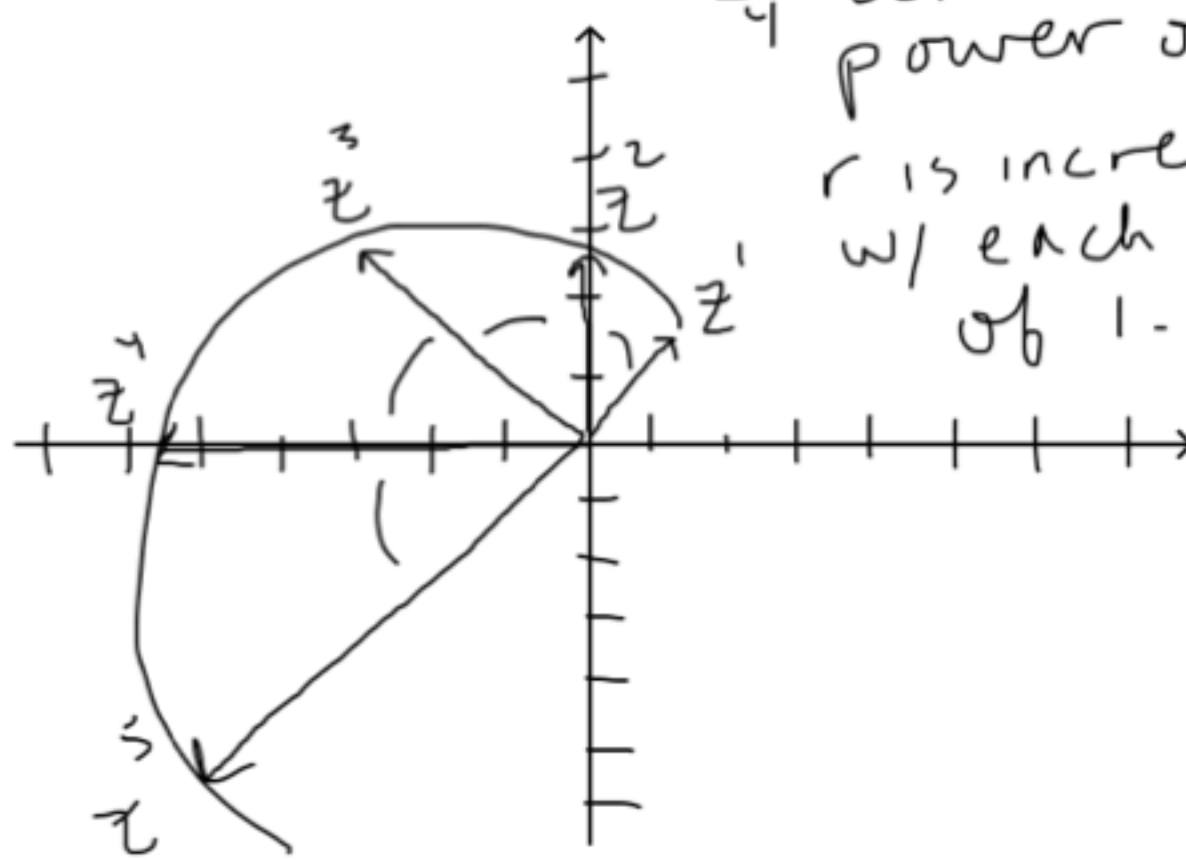
$z^4 = (1.5)^4 \operatorname{cis}(4 \cdot \pi/4) = 5.0625 \operatorname{cis} \pi$

$z^5 = (1.5)^5 \operatorname{cis}(5 \cdot \frac{\pi}{4}) = 7.59375 \operatorname{cis} \frac{5\pi}{4}$

b. Plot these powers of z. What do you notice?

$\frac{\pi}{4}$ between each power of z

r is increasing w/ each power of 1.5



3. Compute $\left(\sqrt{3}-i\right)^4$ and express your answer in rectangular form. (Change the number to polar form, raise to the fourth power then change back to rectangular form. Yes, this is the easiest way.)

① $a+bi \longrightarrow r\,\text{cis}\,\theta = 2\,\text{cis}\,330^\circ$


② $(2\,\text{cis}\,330^\circ)^4 = 2^4\,\text{cis}(4\cdot 330^\circ)$

$= 16\,\text{cis}\,1320^\circ$

$= 16\,\text{cis}\,240^\circ$

③ $r\,\text{cis}\,\theta \longrightarrow a+bi$

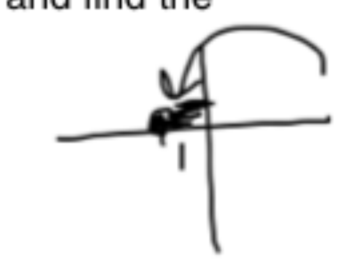
$x = r\cos\theta = 16\cos 240^\circ = 16\left(-\frac{1}{2}\right) = -8$

$y = r\sin\theta = 16\sin 240^\circ = 16\left(-\frac{\sqrt{3}}{2}\right) = -8\sqrt{3}$

$-8 - 8\sqrt{3}\,i$

4. Let z=-1. Use DeMoivre's theorem to show that the positive powers of z are alternately $\pm 1$. Remember that $z=-1=-1+0i$. Change this to polar form and find the following powers of z:

$z = 1\,\text{cis}\,180$




$z^1 = 1\,\text{cis}\,180^\circ = -1$

$z^2 = 1^2\,\text{cis}(2\cdot 180^\circ) = 1\,\text{cis}\,360^\circ = 1$

$z^3 = 1^3\,\text{cis}(3\cdot 180^\circ) = 1\,\text{cis}\,540^\circ = 1\,\text{cis}\,180^\circ = -1$

$z^4 = 1^4\,\text{cis}(4\cdot 180^\circ) = 1\,\text{cis}\,720^\circ = 1$