Precalc Honor
Winkler

10-3 Double Angle Formulas Name_____

1. Find the formula for $\sin(2\alpha)$. HINT: Try finding $\sin(\alpha+\alpha)$.

$\sin(\alpha+\alpha) = \sin\alpha\cos\alpha + \cos\alpha\sin\alpha$

$\sin(2\alpha) = 2\sin\alpha\cos\alpha$

2. Find the formula for $\cos(2\alpha)$. Same hint as above.

$\cos(\alpha+\alpha) = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha$

$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$

$1-\cos^2\alpha$

$= \cos^2\alpha - (1-\cos^2\alpha)$

$\cos(2\alpha) = 2\cos^2\alpha - 1$

$1-\sin^2\alpha$

$= 1-\sin^2\alpha - \sin^2\alpha$

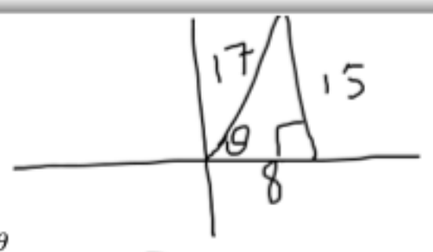
$\cos(2\alpha) = 1 - 2\sin^2\alpha$

3. Find the formula for $\tan(2\alpha)$. You know how.

$\tan(\alpha+\alpha) = \dfrac{\tan\alpha + \tan\alpha}{1-\tan\alpha\tan\alpha}$

$\tan(2\alpha) = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

4. Given that $\cos\theta = \frac{8}{17}$ and $0 < \theta < 90°$, find:



a. cos 2θ

$2\cos^2\theta - 1$

$2\left(\frac{8}{17}\right)^2 - 1$

$2\left(\frac{64}{289}\right) - 1$

$\frac{128}{289} - \frac{289}{289}$

$\frac{-161}{289}$

b. sin 2θ

$2\sin\theta\cos\theta$

$2\left(\frac{15}{17}\right)\left(\frac{8}{17}\right)$

$\frac{240}{289}$

c. tan 2θ

$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{15}{8}\right)}{1-\left(\frac{15}{8}\right)^2} = \frac{\frac{30}{8}}{\frac{64}{64} - \frac{225}{64}}$

$\frac{\frac{30}{8}}{\frac{-161}{64}} = \frac{30}{8} \cdot \frac{-64}{161} = -\frac{240}{161}$

$\sin(2\alpha) = 2\sin\alpha\cos\alpha$

5. Prove the identity: $\sin 4\theta = 4(\sin\theta\cos^3\theta - \cos\theta\sin^3\theta)$

$2\sin 2\theta\cos 2\theta$

$2(2\sin\theta\cos\theta)(\cos^2\theta - \sin^2\theta)$

$4\sin\theta\cos\theta(\cos^2\theta - \sin^2\theta)$

$4(\sin\theta\cos^3\theta - \sin^3\theta\cos\theta)$

leave answer $\surd$

6. Find the exact value of $\cos\frac{\pi}{8}$. Q1

(hint use the double angle formula for cos (2 • π/8)).

$\cos\left(2\cdot\frac{\pi}{8}\right) = 2\cos^2\frac{\pi}{8} - 1$

$\cos\left(\frac{\pi}{4}\right)$

$\frac{\sqrt{2}}{2} = 2\cos^2\frac{\pi}{8} - 1$

$\frac{\sqrt{2}}{2} + 1 = 2\cos^2\frac{\pi}{8}$

$\frac{\sqrt{2}}{4} + \frac{1}{2} = \sqrt{\frac{\sqrt{2}+2}{4}} = \sqrt{\cos^2\frac{\pi}{8}}$

$\frac{\sqrt{\sqrt{2}+2}}{2} = \cos\frac{\pi}{8}$