Math 3H 10-2 Tan $\alpha \pm \beta$ Name:_________

1. Find a counterexample to prove that tan(x+y)=tan x + tan y is not true.

$\tan(45^\circ + 45^\circ)$ $\tan 45^\circ + \tan 45^\circ$

$\tan 90^\circ$ $1 + 1 = 2$

undefined

not same

b. Are there any values that DO work for the above equation?

$x = y = 0 \quad \tan(0+0) = 0$

2. Use the formulas for $\sin \alpha \pm \beta$ to derive the formulas for tan $\alpha \pm \beta$.

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$$

$$= \frac{(\sin\alpha\cos\beta + \cos\alpha\sin\beta)}{(\cos\alpha\cos\beta - \sin\alpha\sin\beta)} \cdot \frac{\left(\frac{1}{\cos\alpha\cos\beta}\right)}{\left(\frac{1}{\cos\alpha\cos\beta}\right)}$$

$$\frac{\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}}{1 - \left(\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}\right)} = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$$

$$\boxed{\begin{aligned} \tan(\alpha+\beta) &= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \\ \tan(\alpha-\beta) &= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} \end{aligned}}$$

3. If you substitute $\alpha = 0$ and $\beta = \theta$ in the subtraction equation above, what property have you proved?

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\tan(0 - \theta) = \frac{\cancel{\tan 0} - \tan\theta}{1 + \cancel{\tan 0 \tan\theta}} = -\tan\theta$$

$$\tan(-\theta)$$

Evaluate each expression without using a calculator.

4. tan 195° $= \tan(150° + 45°)$

$$\frac{\tan 150° + \tan 45°}{1 - \tan 150° \tan 45°}$$

conjugate

$$\frac{-\frac{\sqrt{3}}{3} + 1}{1 - \left(-\frac{\sqrt{3}}{3}\right)(1)} \cdot \frac{3}{3} = \frac{(-\sqrt{3} + 3)}{(3 + \sqrt{3})} \cdot \frac{(3 - \sqrt{3})}{(3 - \sqrt{3})}$$

$$\frac{-3\sqrt{3} + 3 + 9 - 3\sqrt{3}}{9 - 3} = \frac{12 - 6\sqrt{3}}{6} = 2 - \sqrt{3}$$

5. cot (-15°)

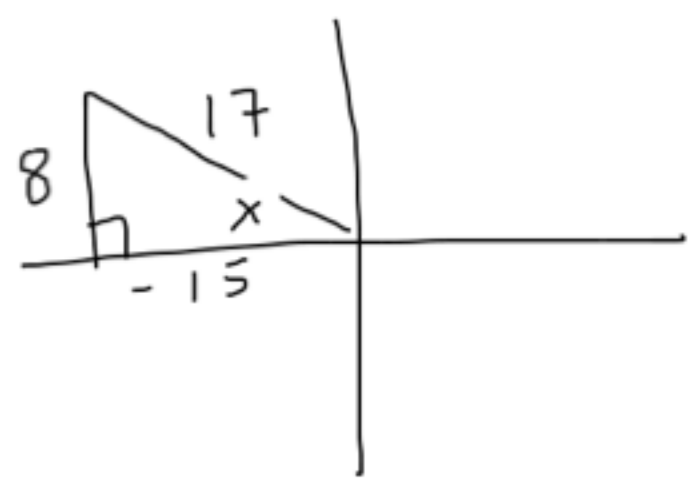
$$\frac{1}{\tan(-15°)} = \frac{1}{\tan(30° - 45°)} = \frac{1}{\frac{\tan 30° - \tan 45°}{1 + \tan 30° \tan 45°}}$$

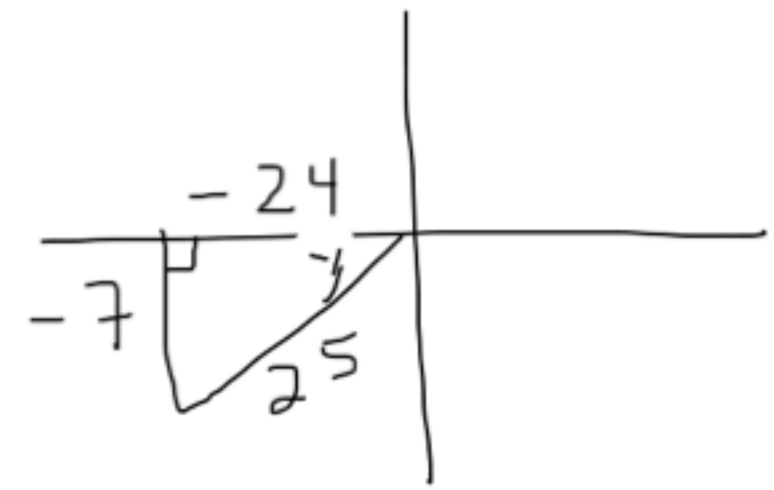
flip: $\dfrac{1 + \tan 30° \tan 45°}{\tan 30° - \tan 45°} = \dfrac{1 + \left(\frac{\sqrt{3}}{3}\right)(1)}{\frac{\sqrt{3}}{3} - 1} \cdot \dfrac{3}{3}$

$$\frac{(3 + \sqrt{3})}{(\sqrt{3} - 3)} \cdot \frac{(\sqrt{3} + 3)}{(\sqrt{3} + 3)} = \frac{12 + 6\sqrt{3}}{-6} = -2 - \sqrt{3}$$

conjugate

6. If π/2 < x < π < y < 3π/2, cos x = -15/17 and tan y = 7/24, find:





A. sin (x+y) $= \sin x \cos y + \cos x \sin y$

$$\left(\frac{8}{17}\right)\left(-\frac{24}{25}\right) + \left(\frac{-15}{17}\right)\left(\frac{-7}{25}\right)$$

$$\frac{-192}{425} + \frac{105}{425} = \frac{-87}{425}$$

B. cos(x-y) $= \cos x \cos y + \sin x \sin y$

$$= \frac{-15}{17} \cdot \frac{-24}{25} + \frac{8}{17} \cdot \frac{-7}{25}$$

$$= \frac{360}{425} + -\frac{56}{425} = \frac{304}{425}$$

C. tan(x+y) $= \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

$$= \frac{-\frac{8}{15} + \frac{7}{24}}{1 - \left(-\frac{8}{15}\right)\left(\frac{7}{24}\right)} \cdot \frac{(15)(24)}{(15)(24)}$$

$$= \frac{(-8)(24) + (7)(15)}{360 + 56} = \frac{-87}{416}$$

D. sin (x-y) $= \sin x \cos y - \cos x \sin y$

$$= \frac{8}{17} \cdot \frac{-24}{25} - \left(\frac{-15}{17}\right)\left(\frac{-7}{25}\right)$$

$$= \frac{-192}{425} - \frac{105}{425} = \frac{-297}{425}$$

E. cos(x+y) $= \cos x \cos y - \sin x \sin y$

$$= \frac{-15}{17} \cdot \frac{-24}{25} - \frac{8}{17} \cdot \frac{-7}{25}$$

$$= \frac{360}{425} + \frac{56}{425} = \frac{416}{425}$$

F. tan(x-y) $= \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

$$= \frac{-\frac{8}{15} - \frac{7}{24}}{1 + \left(-\frac{8}{15}\right)\left(\frac{7}{24}\right)} \cdot \frac{(15)(24)}{(15)(24)}$$

$$= \frac{-192 - 105}{360 - 56} = \frac{-297}{304}$$