

10.2 #29

$$\tan(2\alpha) = \frac{\tan \alpha + \tan \alpha}{1 - \tan \alpha \tan \alpha} = \frac{\frac{1}{5} + \frac{1}{5}}{1 - \frac{1}{5} \cdot \frac{1}{5}} = \frac{\frac{2}{5}}{\frac{24}{25}} = \frac{2}{5} \cdot \frac{25}{24} = \boxed{\frac{5}{12}}$$

$$\tan(4\alpha) = \frac{\tan 2\alpha + \tan 2\alpha}{1 - \tan 2\alpha \tan 2\alpha} = \frac{\frac{5}{12} + \frac{5}{12}}{1 - \frac{5}{12} \cdot \frac{5}{12}} = \frac{\frac{10}{12}}{1 - \frac{25}{144}} = \frac{\frac{10}{12}}{\frac{119}{144}} = \boxed{\frac{120}{119}}$$

$$\tan(4\alpha - \beta) = \frac{\tan 4\alpha - \tan \beta}{1 + \tan 4\alpha \tan \beta} = \frac{\frac{120}{119} - \frac{1}{239}}{1 + \frac{120}{119} \cdot \frac{1}{239}} = \frac{\frac{28561}{28441}}{\frac{28561}{28441}} = \textcircled{1}$$

$$\therefore 4\alpha - \beta = \frac{\pi}{4}$$